ARGOS Location

MEMORANDUM

Subject:	Improving ARGOS Doppler Location Using Kalman Filtering		
Reference:	CLS-DT-MEMO-11-65	**From:**	
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Issue:	1. 0	**To:**	
Date:	02/02/2011	ARGOS Users	
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Abstract

The ARGOS service was launched in 1978 as a joint US-French initiative and is geared towards environmental applications, including oceanography, wildlife tracking, fishing vessel monitoring and maritime safety. The system allows for worldwide, near real-time data collection and positioning of Platform Terminal Transmitters (PTTs). The current ARGOS positioning algorithm is based on a classical non-linear Least Squares estimation technique. We present here a new positioning algorithm that relies on Kalman filtering of the ARGOS frequency measurements. The performance of this new location algorithm are compared to those of the existing algorithm using a large dataset coming from mobiles carrying both an ARGOS transmitter and a GPS receiver. The results show that the new algorithm provides more estimated positions and significantly improves the positioning accuracy. Improvements are most significant for locations obtained in difficult conditions (class A and B locations).

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I. Introduction

The ARGOS service was launched in 1978 as a joint US-French initiative and is geared towards environmental applications, including oceanography, wildlife tracking, fishing vessel monitoring and maritime safety. The system allows for worldwide, near real-time data collection and positioning of Platform Terminal Transmitters (PTTs) using polar low-orbiting satellites of the NOAA and the EUMETSAT. PTTs transmit in the bandwidth of 401.650 MHz ± 30 kHz. Message separation and positioning is achieved by computing the Doppler shift in the carrier frequency of the PTTs seen by a given receiver. To date, over 20,000 platforms are active.

The present processing, based on a classical non-linear Least Squares estimation technique, requires a minimum of two messages during a single satellite pass to compute a geoposition. This technical report focuses on the implementation of a new positioning algorithm based on a multiple-model Kalman filter. Contrary to previous works that use ARGOS fixes as raw data and apply an additional layer to modify or remove them, the new method exploits directly the frequency measurements to compute locations. The advantage of a direct processing is to completely replace the Least Squares analysis by a method able to deliver systematically an error estimate and a geoposition, even when only 1 message is received.

II. ARGOS location

A. Principles

The ARGOS System can locate a platform anywhere on the Earth utilizing the Doppler effect *i.e.* the frequency shift of the signal induced by the relative motion between the satellite and the PTT. Let $\vec{B}$ be the platform location, $\vec{V_B}$ its velocity, $\vec{S}$ the satellite location and $\vec{V_S}$ its velocity in the terrestrial reference frame. The relationship between the transmitting frequency f_t of the platform and the frequency f_r received at the satellite writes as:

$$\begin{aligned} f_r &= h(f_t, \vec{B}, \vec{S}, \vec{V_B}, \vec{V_S}) + v \\ &= f_t \left(1 - \frac{(\vec{V_S} - \vec{V_B}).\vec{u}}{c}\right) + v \end{aligned} \tag{1}$$

with c the speed of light, $\vec{u} = \frac{\vec{S}-\vec{B}}{\|\vec{S}-\vec{B}\|}$ the unitary vector directed from the platform to the satellite and $\|.\|$ the Euclidean norm $\|.\|_2$. The member v is a zero-mean gaussian random noise modeling measurements inaccuracies. The scalar product $(\vec{V_S} - \vec{V_B}).\vec{u}$ accounts for the relative radial velocity between the PTT and the satellite. Since $\left\|\vec{V_B}\right\| \ll \left\|\vec{V_S}\right\|$, the platform speed is neglected in computations.

In the observation function h, the satellite ephemeris $\vec{S}$ and $\vec{V_S}$ are given. The remaining unknowns are the transmitting frequency f_t as well as the platform longitude λ and latitude ϕ. The platform is constrained to be on the terrestrial reference ellipsoid WGS84 which is a standard spheroidal reference surface given by the World Geodetic System [1]. For marine platforms, the altitude is set to 0 while for terrestrial platforms and birds the altitude is given by the digital terrain model GTOPO30 [2].

A major feature of the Doppler location is the existence of two possible positions of the platform that give exactly the same frequency measurements on board the satellite: the *nominal* ("true") location and the *mirror* ("virtual") location. They are symmetrical about the sub-satellite track and, unfortunately, they are not *a priori* distinguishable.

B. The current location algorithm: Least Squares fitting

Least Squares problems arise mostly in data fitting. The aim is to estimate n unknown parameters of an observation model h with $m \geq n$ imperfect measurements. Let $x \in \mathbb{R}^n$ be the vector of parameters and $z \in \mathbb{R}^m$ the vector of observations. For our location problem, x is the vector of $n = 3$ unknowns $(\lambda, \phi, f_t)^T$, $z = (f_1, \ldots, f_m)^T$ is a set of m frequency observations where $(.)^T$ denotes the transpose operator. Importantly, the location and the frequency of the platform are considered fixed during a satellite overpass whose duration can reach 15 min. The Least Squares method can fit problems matching to (1) which can be rewritten as

$$z = h(x) + v \tag{2}$$

where $v \in \mathbb{R}^m$ represents a random gaussian measurement error with mean $\bar{v} = 0$ and variance $R = \sigma^2 I_m$. The sum of the squared residuals is defined as

$$S(x) = \|z - h(x)\|^2 . \tag{3}$$

The Least Squares estimator provides the estimate $\hat{x}$ by minimizing the squared residuals with respect to x.

As the Doppler observation function h is non-linear, the Least Squares estimator has no closed-form solution. To handle non-linearities, the observation function is firstly approximated with a linear one:

$$z = h(x) + v \approx h(\hat{x}_0) + H_0 \delta x_0 + v \tag{4}$$

with $\hat{x}_0$ an initial estimate of the true solution, $\delta x_0 = x - \hat{x}_0$ and $H_0 = \frac{\partial h}{\partial x}\big|_{\hat{x}_0} \in \mathbb{R}^{m \times n}$. The squared residual become

$$S(x) \approx \|z - h(\hat{x}_0) - H_0 \delta x_0\|^2 \tag{5}$$

with minimum

$$\delta \hat{x}_0 = (H_0^T H_0)^{-1} H_0^T \left(z - h(\hat{x}_0)\right) \tag{6}$$

so that the next solution estimation is $\hat{x}_1 = \hat{x}_0 + \delta \hat{x}_0$. The estimate $\hat{x}_1$ is then used as the initial estimate to compute $\hat{x}_2$ and so on. This iterative refinement amounts to the Gauss-Newton Method [3] and is stopped at iteration i when the variation of S becomes sufficiently small or when a maximum number of iterations is reached.

The estimate of the true solution $\hat{x}_0$ in (4) is calculated using the geometric location principle [4, section 3.2]. The geometric location computes two initial estimates: the first close to the nominal location and the second close to the mirror location. The initial transmitting frequency is the last estimated one. The refinement is then successively applied on the two initial estimations using all the messages of the pass and computes two solutions. In order to choose the nominal solution and to check its likelihood, four plausibility tests, described in [4, section 3.2], are applied. If the platform is seen for the first time, there is no transmitting frequency available as initial estimate. So, the algorithm scans the frequency band in steps of 500 Hz around the average frequency of the messages received. For each value, it performs the geometric location and refinement. Of all the solutions computed, it keeps the one with the smallest residual.

In practice, the ARGOS location algorithm adapts the refinement step based on the number of measurements.

1) Calculations with at least 4 measurements: We deal with an *overdetermined* problem because we have more observations that unknowns and Non-Linear Least Squares are directly applicable. An error estimate of $(\lambda, \phi, f_t)^T$ is calculated for both solutions as a covariance matrix

$$cov(x) = \hat{\sigma}_i^2 (H_i^T H_i)^{-1} \tag{7}$$

with the estimated observation noise covariance

$$\hat{\sigma}_i^2 = \frac{S(\hat{x}_i)}{m - n}. \tag{8}$$

2) Calculations with 3 measurements: In this case, $m = n$ and the problem amounts to solve a non-linear equation. As the estimated observation noise of (8) is undefined, the only deliverable information about error is the Geometric Dilution Of Precision (GDOP) in m.Hz^{-1}:

$$GDOP = \sqrt{tr((H_i^T H_i)^{-1}|_{\lambda,\phi})} \tag{9}$$

where $tr(X)$ is the trace of the matrix X and $X|_{\lambda,\phi}$ is the matrix X reduced to its spatial part (λ, ϕ).

3) Calculations with 2 measurements: The system is *underdetermined* with an infinite number of solutions. Hence, we consider the transmitting frequency is known and equal to the last estimated one. We have to solve again a non-linear equation with no estimate error available. For low-power platforms generating mainly 2-message locations, the estimated frequency is not refreshed systematically which contributes also to location inaccuracies.

An interesting point is that the errors of ARGOS location estimates are given as an error ellipse with a semi-major axis a, a semi-minor axis b and an orientation θ (from North when heading East). These quantities are simply derived from the covariance matrix $cov(x)$ available with at least 4 messages. For convenience only, users are also provided with an equivalent error radius R equals to $\sqrt{ab}$ to easily classify locations according to their accuracy. However, users must keep in mind that the error is rarely isotropic so that the error radius only grossly qualifies the error. Use of the error ellipse, instead of the error radius, is strongly recommended.

The locations are estimated with a level of accuracy among 7 Location Classes (LC): 3, 2, 1, 0, A, B and Z. For a satellite pass with a minimum of 4 messages, an error estimate can be computed and is used for the classification. Locations belong to LC 3 with an error radius under 250 m, LC 2 between 250 m and 500 m, LC 1 between 500 m and 1500 m, and LC 0 beyond 1500 m. For a satellite pass with 3 messages and 2 messages, the estimated accuracy is unknown and locations are respectively tagged as LC A and LC B. Locations failing the plausibility tests are tagged as class Z.

C. Limitations of the Least Squares location processing

The current location processing has three main limitations.

1) Noisy trajectories: While positioning errors of 100 m can be reached in best case scenarios, comparisons [5][6][7] with GPS positions showed that many applications do not display such accurate locations. For example, the oscillator's medium term stability is strongly conditioned on the temperature gradients experienced by the PTT during a pass duration of 15 min. These phenomena lead to noisy estimated trajectories. They are amplified when the average number of received messages is low because the estimated transmitting frequency cannot be updated by the location algorithm. Other sources of errors contribute to inaccuracies, such as movement of the PTT between transmissions or ambient and ionospheric noise around the 401.650 MHz frequency.

2) Mirror locations: The location algorithm provides two solutions where the first one is the most plausible and considered as the nominal location. Sometimes, the algorithm makes the wrong choice and gives the mirror location as first solution. Mirror locations can reach an error of 5000 km which is the diameter of the visibility circle of the satellites.

3) No error estimation for locations with 3 or 2 messages: When the number of messages received by the satellite is less than 4, no error estimation is given. This includes locations tagged as A and B, and results in the employment of heterogeneous data by the users.

III. The new location algorithm: multiple-model Kalman Filtering

Notations are standard. $P(.)$, $p(.)$ and $E[.]$ respectively represent a probability, a probability density function (pdf), and an expectation. $\mathcal{N}(x;\bar{x},P)$ stands for the real Gaussian distribution with mean $\bar{x}$ and covariance P.

A. State-space representation

The location problem can be formulated under a state-space representation. The true state x of the platform, including its location and transmitting frequency, follows a hidden discrete-time Markov process described by a transition model:

$$x_k = f_k(x_{k-1}) + w_k \tag{10}$$

with k the time index of the satellite pass, f_k the state-transition function and $w_k \sim \mathcal{N}(w_k;\bar{w}_k,Q_k)$ the process noise. The Doppler observation model yields the observed frequency measurements z from the true state x:

$$z_k = h_k(x_k) + v_k \tag{11}$$

with h_k the Doppler observation function and $v_k \sim \mathcal{N}(v_k;\bar{v}_k,R_k)$ the measurement noise. For (10) and (11), the sequences $\{h_.,\bar{v}_.,R_.\}$ and $\{f_.,\bar{w}_.,Q_.\}$ are given, and the noises w_k and v_k are independent of x_{k-1}.

B. The Kalman filter applied to Argos location

The aim is to approximate over time the posterior pdf $p(x_k|z_{1:k})$ of the base state x_k conditioned by the observed measurements $z_{1:k} = (z_1,\ldots,z_k)$. The Kalman filter [8] is a Bayesian estimator that computes analytically estimates of the true state assuming all pdfs are normally distributed. This way, $p(x_k|z_{1:k})$ can be reduced to its first two moments: the posterior mean $\hat{x}_{k|k} = E[x_k|z_{1:k}]$ (equivalently the location estimate of the platform) and the posterior covariance $P_{k|k} = E[(x_k - \hat{x}_{k|k})^T(x_k - \hat{x}_{k|k})|z_{1:k}]$ (equivalently the location error) of the state x. The Kalman filter first propagates forward in time the last state estimate $\hat{x}_{k-1|k-1}$ and the last covariance estimate $P_{k-1|k-1}$ using only the transition model. The predicted state $\hat{x}_{k|k-1}$ and covariance $P_{k|k-1}$ are finally corrected by the acquired measurements to build $\hat{x}_{k|k}$ and $P_{k|k}$. The strength of the correction step depends on the filter gain which weights the predicted estimate and the measurements by their relative errors. The Kalman filter is an optimal mean-square estimator as it minimizes the expectation $E\left[||x_k - \hat{x}_{k|k}||^2\right] = tr(P_{k|k})$. The estimation is done sequentially meaning frequency observations are to be employed as though they are arriving in real time. For a given satellite overpass at time k, we compute a unique location. In other words, measurements are batch-wise processed so that z_k is a vector of observed frequencies during the pass.

The Kalman filter technique has proved particularly fruitful in target tracking for space and military applications. This algorithm was originally designed for linear systems but has been extensively developed until the last decade [9] [10] to handle non-linearities resulting in the so-called Square Root Unscented Kalman filter (SRUKF) [11]. The SRUKF, which is used here, addresses more efficiently the issues of non-linear transition and observation models than the wide spread Extended Kalman Filter. Rather than calculating the Jacobians of functions to build a linear approximation, the SRUKF performs a statistical linearization based on a sigma-point transformation [12].

Compared with Least Squares fitting, calculations can be made regardless of the number of measurements and produces systematically an error estimate through the covariance $P_{k|k}$. Concretely, locations calculated with 3 and 2 messages (tagged respectively as class A and B) have an error estimate as an error ellipse and 1-message locations can now be computed (tagged as class B too). The filter output is directly a single nominal solution. Moreover, the filter does not assume the frequency remains identical between two locations when less than three messages are recorded. The frequency value is always refreshed to handle its variations even if the average number of messages is low. As previously, the location is tested in order to check its likelihood before being distributed to ARGOS users.

C. Choosing a dynamic model for ARGOS Location

Kalman filtering requires specification of how position at time k relates to position at time $k+1$. As ARGOS is used to track a wide variety of mobiles, one cannot be very specific about that relation (the so-called dynamical model) and the simplest choice is to use a random walk model. The dynamical model thus reads:

$$x_k = x_{k-1} + w_k = \begin{pmatrix} \lambda_{k-1} \\ \phi_{k-1} \\ f_{t,k-1} \end{pmatrix} + w_k \tag{12}$$

with

$$\bar{w}_k = 0,\ Q_k = \begin{pmatrix} 2D_\lambda \Delta t_k & 0 & 0 \\ 0 & 2D_\phi \Delta t_k & 0 \\ 0 & 0 & V_f \end{pmatrix}. \tag{13}$$

The diffusion parameters of the random walk $D_\lambda = D_\phi = D$ in m^2.s^{-1} is a function of V_{max}, the maximum velocity of the platform. The value $2D\Delta t_k$ is equal to the area of the 1-sigma probability surface of the next platform location x. It increases linearly with the elapsed time $\Delta t_k = t_k - t_{k-1}$ since the previous satellite overpass. Notice that the transmitting frequency does not follow a random walk. We consider the frequency remains on average identical between two locations but can evolve with a stationary noise $V_f = 100$ Hz^2 to handle temperature gradients. Finally, the noise on each frequency measurement is set to 0.4 Hz, a crude but realistic assumption.

Directed movements of the platforms are well modeled with a correlated random walk (CRW) which is a random walk on the velocity. The prediction step writes as

$$x_k = \begin{pmatrix} 1 & 0 & \Delta t_k & 0 & 0 \\ 0 & 1 & 0 & \Delta t_k & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} x_{k-1} + w_k, \tag{14}$$

$$\bar{w}_k = 0,\ Q_k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2D'_\lambda \Delta t_k & 0 & 0 \\ 0 & 0 & 0 & 2D'_\phi \Delta t_k & 0 \\ 0 & 0 & 0 & 0 & V_f \end{pmatrix}$$

assuming $x = (\lambda, \phi, v_\lambda, v_\phi, f_t)^T$. This model predicts the next position thanks to the last estimation of the platform velocity. Note the process noise, with diffusion parameters D'_λ and D'_ϕ, is only applied to the two components of the velocity.

Of course, a single random-walk model cannot perfectly reproduce all platform behaviors and a better modeling approach would be to admit mobiles can switch between multiple behaviors: a ship may be conducting fishing operations then steam back to port, while a marine mammal may reside in wintering grounds for several months before migrating to new locations. As such, observed velocities may vary by an order of magnitude along the track, inducing changes in the temporal correlation and breakage of the Markovian property in the single model approach.

Another reason to employ several models is that with a single random walk model the next computed location tends to be attracted towards the last one. Indeed, the main hypothesis built into the random walk model is that the expected position of the platform at time $k+1$ is that at time k. This assumption has the merit of making impossible the large moves that would take the estimated position towards the mirror position and, more generally, to reduce the position estimation noise. However, it can create a tendency to underestimate the amplitude of the movement between k and $k+1$. This tendency shall be most noticeable when the platform movement is far from a random walk and when measurements are lacking, or have large errors, and are thus little weighted in the position estimation process. This is typically the case when attempting to locate platforms with very directed movements (as opposed to random movements), using low quality measurements (which occurs when only a few messages per pass are received, when the pass geometry is unfavorable and/or when the signal transmitted is weak or corrupted). We are thus naturally led to consider the use of a (limited) set of dynamical models.

D. A bank of Kalman filters: multiple-model approach

Multiple model approach is suitable for tracking multiple hypotheses, behaviors or *modes* when filtering the data. We consider a set of modes $\mathscr{M}$ of cardinality M. At time k, the mode in effect is represented by the discrete random index m_k. The sequence of random variables $m_0, m_1, m_2, \dots$ follows an homogeneous finite-state Markov chain with

$$\forall (i,j) \in \mathscr{M} \times \mathscr{M},\ P(m_k = i | m_{k-1} = j) = \pi_{ji}. \tag{15}$$

At initial time, the distribution of x_0 is characterized by $p(x_0) = \sum_{m_0 \in \mathscr{M}} P(m_0)p(x_0|m_0)$, with

$$P(m_0 = i) = \mu_0^{(i)},\ p(x_0|m_0 = i) = \mathscr{N}(x_0; \hat{x}_{0|0}^{(i)}, P_{0|0}^{(i)}), \tag{16}$$

and the statistics $\mu_0^{(i)}, \hat{x}_{0|0}^{(i)}, P_{0|0}^{(i)}$ are given. The transition pdf between times $k-1$ and k satisfies $p(m_k, x_k|m_{k-1}, x_{k-1}) = P(m_k|m_{k-1})p(x_k|m_k, x_{k-1})$.

Conditionally on the active mode m_k between times $k-1$ and k, the assumed base state dynamics $p(x_k|m_k = i, x_{k-1})$ is described by the state equation

$$x_k = f_k^{(i)}(x_{k-1}) + w_k^{(i)},\ w_k^{(i)} \sim \mathscr{N}(w_k^{(i)}; \bar{w}_k^{(i)}, Q_k^{(i)}), \tag{17}$$

where the dynamics noise $w_k^{(i)}$ is white and independent of $x_0^{(i)}$, and $\{f_\cdot^{(i)}, \bar{w}_\cdot^{(i)}, Q_\cdot^{(i)}\}_{i \in \{1,\ldots,M\}}$ are given. Similarly, the observation pdf $p(z_k|m_k = i, x_k)$, which unites the state vector and the measurement z_k under the assumption that $m_k = i$, straightly comes from the linear output equation

$$z_k = h_k^{(i)}(x_k) + v_k^{(i)},\ v_k^{(i)} \sim \mathscr{N}(v_k^{(i)}; \bar{v}_k^{(i)}, R_k^{(i)}), \tag{18}$$

where the measurement noise $v_k^{(i)}$ is white and independent of $x_0^{(i)}, \{w_l^{(i)}\}_{l \in \{1,\ldots,k-1\}}$, and $\{h_\cdot^{(i)}, \bar{v}_\cdot^{(i)}, R_\cdot^{(i)}\}_{i \in \{1,\ldots,M\}}$ are given.

In the framework of discrete-time jump Markov systems, the probability density function $p(x_k|z_{1:k})$ writes as a Gaussian mixture with M^{k+1} components [13]:

$$p(x_k|z_{1:k}) = \sum_{i_{0:k} \in \mathscr{M}^{k+1}} p(x_k|m_{0:k} = i_{0:k}, z_{1:k})P(m_{0:k} = i_{0:k}|z_{1:k}) \tag{19}$$

where $m_{0:k} = \{m_0, \ldots, m_k\} = i_{0:k} = \{i_0, \ldots, i_k\}$ is the model sequence from time 0 to k. The exponentially growing complexity of the problem precludes an exact resolution. The Interacting Multiple Model (IMM) algorithm has become a standard approach to derive a tractable solution by merging the growing tree of model sequences. On the basis of the sequence of measurements $z_{1:k} = (z_1, \ldots, z_k)$, the IMM propagates along time an approximation of the posterior density $p(x_k|m_k, z_{1:k})$ and the posterior mode probability $P(m_k|z_{1:k})$. The IMM filter, described in Algorithm 1 for the Gaussian case with linear Kalman filters to simplify, was first outlined in [14] and is surveyed with multiple-model algorithms in [13].

Our wish is to make a random walk with state vector $x = (\lambda, \phi, f_t)^T$ and a correlated random walk with $x = (\lambda, \phi, v_\lambda, v_\phi, f_t)^T$ cooperate inside an IMM filter. Nevertheless, the standard IMM is not fully adapted to handle a bank of mode-matched filters with state vectors of heterogeneous size and meaning. To circumvent the use of heterogeneous-order state-space models, only the prediction step of the CRW is retained to build a second model. The velocity $\hat{v}_k = (\hat{v}_\lambda, \hat{v}_\phi)^T$ that will be used in the prediction step at time $k+1$ is computed separately with an exponential moving average such as

$$\hat{v}_k = \alpha \tilde{v}_k + (1-\alpha)\hat{v}_{k-1} \tag{20}$$

with $\alpha = 0.3$ and $\tilde{v}_k$ an estimation of the velocity by the finite difference formula

$$\tilde{v}_k = \frac{(\lambda_k, \phi_k)^T - (\lambda_{k-1}, \phi_{k-1})^T}{\Delta t_k}. \tag{21}$$

With this value of α, the weight of the 5 most recent velocity observations used by the exponential moving average is about 86% of the total weight. This parameter does not depend on time so that the velocity estimation does not depend on the duty cycle of the PTT. The velocity $\hat{v}_k$ is smoothed to lower the erratic behavior of the finite difference estimation that is generally observed on raw ARGOS data. Ultimately, the state equation, with $x = (\lambda, \phi, f_t)^T$, is

$$x_k = x_{k-1} + \begin{pmatrix} \hat{v}_{k-1}\Delta t_k \\ 0 \end{pmatrix} + w_k, \tag{22}$$

$$\bar{w}_k = 0,\ Q_k = \begin{pmatrix} 2D_\lambda \Delta t_k & 0 & 0 \\ 0 & 2D_\phi \Delta t_k & 0 \\ 0 & 0 & V_f \end{pmatrix}.$$

It amounts to a random walk with a time-dependent bias term. Contrary to the random walk, the model described above is not linear. During directed movement, the IMM is expected to switch to this additional mode.

Algorithm 1 IMM FOR LINEAR GAUSSIAN CASE

1: Transition probability (given): $\pi_{ji} = P(m_k = i|m_{k-1} = j)$
2: **IF** $k = 0$ **THEN**
3: **FOR** $i \in \mathscr{M}$ **DO**
4: Determine initial mode probability: $\mu_0^{(i)}$
5: Determine initial estimate and covariance: $\hat{x}_{0|0}^{(i)}$ and $P_{0|0}^{(i)}$.
6: **END FOR**
7: **END IF**
8: **IF** $k \geq 1$ **THEN**
9: **FOR** $i \in \mathscr{M}$ **DO** {Mode-conditioned reinitialization}
10: Predicted mode probability: $\mu_{k|k-1}^{(i)} = P(m_k = i|z_{1:k-1}) = \sum_{j \in \mathscr{M}} \pi_{ji}\mu_{k-1}^{(j)}$
11: Mixing weight: $\mu_{k-1}^{j|i} = P(m_{k-1} = j|m_k = i, z_{1:k-1}) = \pi_{ji}\mu_{k-1}^{(j)} / \mu_{k|k-1}^{(i)}$
12: Mixing estimate: $\bar{x}_{k-1|k-1}^{(i)} = E[x_{k-1}|m_k = i, z_{1:k-1}] = \sum_{j \in \mathscr{M}} \mu_{k-1}^{j|i}\hat{x}_{k-1|k-1}^{(j)}$
13: Mixing covariance: $\bar{P}_{k-1|k-1}^{(i)} = \sum_{j \in \mathscr{M}} \mu_{k-1}^{j|i}\left[P_{k-1|k-1}^{(j)} + \left(\hat{x}_{k-1|k-1}^{(j)} - \bar{x}_{k-1|k-1}^{(i)}\right)\left(\hat{x}_{k-1|k-1}^{(j)} - \bar{x}_{k-1|k-1}^{(i)}\right)^T\right]$
14: **END FOR**
15: **FOR** $i \in \mathscr{M}$ **DO** {Mode-matched Kalman filtering (linear equations)}
16: Predicted state and covariance: $\hat{x}_{k|k-1}^{(i)} = F_{k-1}^{(i)}\bar{x}_{k-1|k-1}^{(i)} + \bar{w}_{k-1}^{(i)}$ and $P_{k|k-1}^{(i)} = F_{k-1}^{(i)}\bar{P}_{k-1|k-1}^{(i)}\left(F_{k-1}^{(i)}\right)^T + Q_{k-1}^{(i)}$
17: Measurement residual, residual covariance: $\widetilde{z}_k^{(i)} = z_k - H_k^{(i)}\hat{x}_{k|k-1}^{(i)} - \bar{v}_k^{(i)}$ and $S_k^{(i)} = H_k^{(i)}P_{k|k-1}^{(i)}\left(H_k^{(i)}\right)^T + R_k^{(i)}$
18: Updated state and covariance: $\hat{x}_{k|k}^{(i)} = \hat{x}_{k|k-1}^{(i)} + K_k^{(i)}\widetilde{z}_k^{(i)}$ and $P_{k|k}^{(i)} = P_{k|k-1}^{(i)} - K_k^{(i)}S_k^{(i)}\left(K_k^{(i)}\right)^T$ with the filter gain $K_k^{(i)} = P_{k|k-1}^{(i)}\left(H_k^{(i)}\right)^T\left(S_k^{(i)}\right)^{-1}$
19: **END FOR**
20: **FOR** $i \in \mathscr{M}$ **DO** {Posterior mode probability update}
21: Mode likelihood: $L_k^{(i)} = p(\widetilde{z}_k^{(i)}|m_k = i, z_{1:k-1}) = \mathscr{N}\left(\widetilde{z}_k^{(i)}; 0, S_k^{(i)}\right)$
22: Posterior mode probability: $\mu_k^{(i)} = P(m_k = i|z_{1:k}) = \frac{\mu_{k|k-1}^{(i)}L_k^{(i)}}{\sum_{j \in \mathscr{M}} \mu_{k|k-1}^{(j)}L_k^{(j)}}$
23: **END FOR**
24: Compute the overall estimate $\hat{x}_{k|k}$ and covariance $P_{k|k}$ of x: $\hat{x}_{k|k} = \hat{x}_{k|k}^{(\hat{m}_k)}$ and $P_{k|k} = P_{k|k}^{(\hat{m}_k)}$ with $\hat{m}_k = \arg\max_{m_k=1,\ldots,M} P(m_k|z_{1:k})$.
25: **END IF**

As explained above, the IMM filter computes over time an approximation of $p(x_k|m_k, z_{1:k}) = \mathscr{N}(x_k; \hat{x}_{k|k}^{(i)}, P_{k|k}^{(i)})$ and $P(m_k|z_{1:k})$. For output processing purpose, the multi-model estimate and covariance provided to users are the posterior mean $\hat{x}_{k|k}^{(\hat{m}_k)}$ and covariance $P_{k|k}^{(\hat{m}_k)}$ associated with the most probable a posteriori mode estimate

$$\hat{m}_k = \underset{m_k=1,\ldots,M}{\arg\max}\, P(m_k|z_{1:k}). \tag{23}$$

Transition prior probabilities are set to $\pi_{ji} = P(m_k = i|m_{k-1} = j) = 0.5\ \forall(i,j) \in \mathscr{M} \times \mathscr{M}$. Thus, the predicted mode probability of step 10 in Algorithm 1 is systematically equal to 0.5. This implies that mode-matched estimates $\{\hat{x}_{k|k}^{(i)}\}_{i=1,\ldots,M}$ are selected according to the values of the likelihoods $\{L_k^{(i)}\}_{i=1,\ldots,M}$ calculated in the bank of Kalman filters because the posterior mode probability of step 22 rewrites as

$$\mu_k^{(i)} = \frac{L_k^{(i)}}{\sum_{j \in \mathscr{M}} L_k^{(j)}}. \tag{24}$$

In other words, the past of the model sequence does not influence the current choice of the output estimate. Nevertheless, the mode-conditioned reinitialization that computes the mixing estimates and covariances at the input of the bank of Kalman filters depends on the last known mode probabilities.

E. Dealing with location ambiguity

Kalman filtering is a recursive process that needs a start point $\hat{x}_{0|0}$ and a covariance $P_{0|0}$. For this, a Least Squares analysis is performed to compute the initial state and covariance with at least four measurements. However, the Least Squares filter systematically produces a nominal and mirror location that we are not able to distinguish at the first satellite overpass. This ambiguity is solved with the next pass by using successively the Kalman filter on both locations and by choosing the most likely one. The selection criterion is the value of the residual likelihood. The residual $\widetilde{z}_k$ is the difference between the true measurements and those computed from the predicted location $\hat{x}_{k|k-1}$:

$$\widetilde{z}_k = z_k - h_k(\hat{x}_{k|k-1}) - \bar{v}_k \tag{25}$$

with $S_k = cov(\tilde{z}_k)$ the residual covariance. Knowing that the residual follows a normal distribution $\mathcal{N}(\tilde{z}_k; 0, S_k)$, we pick up the location associated to the most probable one. To date, we never encountered a case where the ambiguity resolution fails provided frequency measurements are correct.

To handle tricky cases such as discontinuities, transmitting frequency jumps, wrong ambiguity resolution or fast platform transportation, the algorithm is able to detect them through the plausibility checks and reinitializes itself. More generally, when any location is invalidated by the plausibility checks, the algorithm attempts an automatic and instantaneous reset using a Least Squares analysis so that the filtering continues seamlessly in the next satellite overpass.

Although the Kalman filter provides only one location, it is legitimate to ask why it is always the nominal location. With the traditional Least Squares method, it is possible to remove completely the mirror location by mixing measurements from several satellite passes. This requires gathering frequency measurements close in time, *i.e.* included in half an hour maximum. Beyond 30 min, the calculation is hazardous because the platform may move significantly. From a geometrical point of view, the platform has to be in the visibility circle of several satellites which is an event of frequency between 0% (at the equator) and 20% (at the poles). The existence of at least two points of view removes the mirror solution. In comparison, the Kalman filter introduces a direct link between locations with the transition model. The effect of the model is twofold: it creates a mathematical relationship between two successive satellite passes and it takes the possible displacement of the platform into account. Thus, the point of view of the last pass is shared through the last location estimate and, mainly, through the covariance matrix. Additionally, the movement model modifies them depending the elapsed time. The shape of the error ellipse (flatness and orientation), derived from the covariance matrix, is indeed highly constrained by the relative position of the satellite and the platform. So, the Kalman filter recreates the conditions of multi-pass processing.

IV. Validation of the new location algorithm

A. The test dataset

1) Data origin: Location error comparisons between the current technique “Least Squares” (LS) and the next generation “Kalman Filter” (KF) were performed on an exhaustive sample including 65 birds (marabouts and geeses), 23 land animals (gnus and bighorns), 67 marine animals (turtles, elephant seals and sea lions), 23 ships and 50 drifters. Each platform is equipped with a GPS receiver so that the true location is precisely known. To date 228 beacons and around 180 000 satellite passes have been collected. The aim is to cover a wide range of behaviors, PTT types and location areas. The Table I presents in detail the platform datasets.

2) Argos data processing: For each platform, satellite overpasses were built by collecting all the frequency measurements of the emitting period. Satellite ephemeris were fetched from our database and were interpolated at the date of the measurements using an Everett interpolation. As the accuracy of the ephemeris is refined at the end of each day and this analysis is done several months or years later, their quality is the best possible. Frequency measurements and the associated ephemeris are the raw input data of the two compared algorithms and are processed in chronological order. It amounts to a off-line processing and allows to get optimal, repeatable and comparable location results. The output is the ARGOS locations that will be compared to GPS fixes. The value of V_{max} used in the random walk models is equal to the default value stored in our database for each application. The quantity of locations collected for each dataset is presented in Table I.

In other words, we do not use ARGOS locations that are real-time computed by the Least Squares analysis because we are not able to simulate real-time processing behavior for both algorithms. The order in which data were processed is unknown and the original satellite ephemeris are not available as they are replaced with more accurate ones at the end of the day.

3) GPS data processing: When possible, GPS tracks are decoded at CLS otherwise they were directly provided by users. GPS fixes are considered as the true locations of the platforms and thus the following accuracy estimations do not take GPS errors into account.

B. Comparing GPS and ARGOS locations

Locations invalidated by the algorithms (tagged as class Z) were removed and only periods after the deployment of the beacons are taken into account. As GPS location dates do not match exactly ARGOS location dates, we linearly interpolate GPS tracks to the dates of ARGOS locations. The error is the distance between an ARGOS location and the corresponding interpolated GPS fixe. If an ARGOS location is too far in time from a GPS location, the interpolation can disturb error calculation. To avoid this, we keep only locations that are less than a chosen duration from a GPS fixe. The number of locations selected for error analysis and the maximum time gap with the nearest GPS location are given in Table II. The time gap is set to 15 min except for the two data sets concerning generally slow moving terrestrial animals (gnu and bighorn) for which the maximum allowed time gap is 60 minutes. Note most error analysis [5][6][7] performed on ARGOS-GPS platforms include the

TABLE I

NUMBER OF PLATFORMS AND NUMBER OF ARGOS LOCATIONS COLLECTED AFTER CALCULATIONS AND VALIDATED BY THE ALGORITHMS

	Number of platforms	Number of validated locations	
		Least Squares	Kalman
Marabout	5	3472	3800
Geese	60	17610	25545
Gnu	10	2029	2204
Bighorn	13	1819	2211
Flatback Turtle	19	16879	24208
Green Turtle	21	8690	16003
Galapagos Sea Lion	8	1224	1680
Elephant Seal	19	4593	10038
Ship	23	20160	23377
Drifter	50	62617	70384
Total	**228**	**139093**	**179450**

removal of mirror locations or the use of destructive filters based *e.g.* on the maximum velocity of the platform. As we are interested in working on the most raw platform tracks, no additional filtering or removal operations are performed.

TABLE II

MAXIMUM TIME GAP WITH THE NEAREST GPS LOCATION AND NUMBERS OF ARGOS LOCATIONS SELECTED FOR ERROR CALCULATIONS

	Max GPS Gap (min)	Number of selected locations	
		Least Squares	Kalman
Marabout	15	431	466
Geese	15	4289	5263
Gnu	60	437	460
Bighorn	60	401	445
Flatback Turtle	15	3364	4896
Green Turtle	15	2375	3567
Galapagos Sea Lion	15	383	493
Elephant Seal	15	2346	5152
Ship	15	17356	19611
Drifter	15	54749	61252
Total		**86131**	**101605**

TABLE III

VARIATIONS OF THE NUMBER OF COMPUTED LOCATIONS (EXCEPT 1-MESSAGE LOCATIONS)

	4 messages or more (%)	2 or 3 messages (%)
Marabout	3.8	1.7
Geese	8.2	12.5
Gnu	2.9	0.3
Bighorn	12.7	2.9
Flatback Turtle	6.4	6.4
Green Turtle	1.8	3.1
Galapagos Sea Lion	2.5	2.8
Elephant Seal	5.1	8.3
Ship	2.5	2.1
Drifter	2.4	2.7

Three main points are examined to compare the performances of the Least Squares and the Kalman calculation processing.

1) Validated location number: We measure the quantity of validated locations distributed to users. A location is validated if it passed the plausibility checks and is tagged with classes 0, 1, 2, 3, A or B. The Table III gives the amount of additional locations computed by the Kalman filter taking the Least Squares method as reference. The 1-message locations are ignored because they are not computed with the Least Squares method.

2) Cumulative frequency distribution: The cumulative frequency distribution of the error is computed. Two statistics of the distribution are especially monitored: the mean error and the standard deviation. Contrary to the median error, the mean and the standard deviation are sensitive to outlier locations, such as mirror locations or GPS fixes with bit errors. To reduce the influence of outliers in the sample, only the values below the 95^{th} percentile of the error distribution are taken into account for both algorithms.

The common approach to analyze ARGOS data is to split locations according to their location class (3, 2, 1, 0, A, B) and to perform the error analysis for each class. As we explained before, the classification, based on the equivalent radius, is convenient but mathematically approximate because the error is not isotropic. Moreover, LC A and B are distinguished by the number of messages received (respectively 3 and 2) whereas LC 3, 2, 1 and 0 are distinguished by the equivalent error radius. This distinction is no longer valid as the estimated accuracy is now always given. Therefore, locations are analyzed only in terms of quantity of information, *i.e.* the number of messages received. The errors are computed for 4 messages or more (overdetermined system) and for less than 4 messages. The 1-message locations are handled separately as they are new with the Kalman filter.

The mean errors are presented in Table IV and the standard deviations in Table V considering a bank models including both a random walk and a biased random walk. Variation of the errors are indicated taking the Least Squares errors as reference. The errors of 1-message locations are given separately in Table VI. As these kind of locations are not computed with the Least Squares method, comparisons are done with locations calculated with 2 or 3 messages.

3) Median tangential errors: The errors parallel to the platform trajectory are estimated. They are computed by projecting a vector directed from the interpolated GPS location to the corresponding ARGOS location on the axis parallel to trajectory. This axis is oriented as the velocity vector. They aim at detecting a systematic bias along the track and are compared to the Least Squares processing which cannot be biased. The median of theses errors is given for a single model of random walk in Table VII and for a bank of models including both a random walk and a biased random walk in Table VIII. A negative value may indicate a systematic lag and a positive value, an advance.

C. Results and discussion

The mean errors are reduced by several tens of percent: between 0 % and 32 % with more than 4 messages and between 26 % and 76 % with 3 or 2 messages. In the same vein, the standard deviations of errors decrease up to 47 % with a minimum of 4 messages, up to 79 % with 3 or 2 messages. Thus, the Kalman filter approach contributes to a better accuracy and a lower dispersion of the errors, meaning fewer outliers. The improvement is even greater when the number of messages is low. The lack of information is indeed now compensated by the introduction of the last known location in the calculations. For 1-message locations, the mean errors and the standard deviations are mainly under 10 km except for the two bird datasets. This order of magnitude is comparable to the locations computed with 2 and 3 messages with the Least Squares Analysis.

The amount of additional locations, excluding 1-message passes, varies from 0.3 % to 12.7 %. There are two origins of these new locations distributed to users:

- The geometric location performed by the Least Squares method upstream of the refinement step can fail when the system to solve is ill-conditioned and is prevented from distributing a location. These case occur for example when the platform is close to the sub-satellite track.
- The overall accuracy improvements give more coherent locations and it is legitimate to validate an increased number.

The majority of additional fixes comes from the 1-message locations. For PTTs with failing batteries, solar cells having difficulty charging or animals occupying areas of intense industrial noise, these new locations may be of high biological value despite their lower overall quality.

The median tangential errors show how the multiple-model framework can improve the processing. With only one model of random walk, the median is close to zero when at least 4 messages are available. Nevertheless, this error can be as high several kilometers with less messages. The most emblematic example is the elephant seal dataset because these animals are located with 2 messages on average and they have actually a steady directed movement. With few messages, the weight of the corrective step is weak and the algorithm relies mainly on the prediction step, *i.e.* the transition model. The transition model of the random walk hypothesizes the platform does not move in average. With the combination of two models, including one adapted to directed movements, these tangential errors are on average under 10 % of the mean errors. It is acceptable as the majority of the inaccuracies is due to the satellite orbit error and the measurement noise.

V. Conclusions and Future Work

The improvements of the new processing are:

- Over the total number of locations tested, the mean error is reduced between 10 % and 65 %. The overall dispersion decreases between 14 % and 83 % so that the majority of outlier locations are corrected. The new 1-message locations have an accuracy comparable to Least Squares with less than 4 messages.
- The number of location distributed raises between 0.3 % and 12.7 %, excluding 1-message passes.

TABLE IV

MEAN ERRORS (EXCEPT 1-MESSAGE LOCATIONS) AND THEIR VARIATIONS

	4 messages or more			2 and 3 messages			Total		
	LS (m)	KF (m)	**Var. (%)**	LS (m)	KF (m)	**Var. (%)**	LS (m)	KF (m)	**Var. (%)**
Marabout	2756	2620	**-5**	8002	5912	**-26**	3305	2961	**-10**
Geese	3199	3239	**1**	17022	9183	**-46**	5970	4640	**-22**
Gnu	559	543	**-3**	4468	1060	**-76**	647	580	**-10**
Bighorn	530	471	**-11**	2637	1335	**-49**	884	598	**-32**
Flatback Turtle	2284	1648	**-28**	5569	2239	**-60**	4543	2066	**-55**
Green Turtle	727	614	**-16**	3340	1752	**-48**	2237	1309	**-42**
Galapagos Sea Lion	1345	1287	**-4**	5744	2895	**-50**	3214	2018	**-37**
Elephant Seal	2689	2056	**-24**	18957	5595	**-70**	14845	5003	**-65**
Ship	1445	986	**-32**	3854	1941	**-50**	2007	1229	**-39**
Drifter	456	370	**-19**	1201	792	**-34**	620	469	**-24**

TABLE V

STANDARD DEVIATIONS OF ERRORS (EXCEPT 1-MESSAGE LOCATIONS) AND THEIR VARIATIONS

	4 messages or more			2 and 3 messages			Total		
	LS (m)	KF (m)	**Var. (%)**	LS (m)	KF (m)	**Var. (%)**	LS (m)	KF (m)	**Var. (%)**
Marabout	3029	2764	**-9**	6420	5790	**-10**	3684	3175	**-14**
Geese	3766	3657	**-3**	23103	10352	**-55**	8670	5532	**-36**
Gnu	434	430	**-1**	6316	722	**-89**	571	460	**-19**
Bighorn	642	506	**-21**	3499	1840	**-47**	1233	709	**-42**
Flatback Turtle	2732	1443	**-47**	6350	1735	**-73**	5481	1667	**-70**
Green Turtle	585	497	**-15**	4150	1597	**-62**	3024	1299	**-57**
Galapagos Sea Lion	1763	1609	**-9**	5894	2813	**-52**	4227	2346	**-44**
Elephant Seal	3159	1932	**-39**	36214	4795	**-87**	27006	4481	**-83**
Ship	1444	880	**-39**	4273	1850	**-57**	2158	1156	**-46**
Drifter	358	267	**-25**	1134	620	**-45**	579	384	**-34**

- The mirror locations are eliminated except for the initialization stage of the algorithm.
- For A and B location classes, the estimated error is provided.

The most obvious difficulty of the filtering approach developed here is to choose a dynamical model usable for all platforms. However we showed it is possible to improve the overall accuracy combining a limited set of models

Future work will concentrate on improving the IMM approach following [15]. Such improvements shall be included in forthcoming versions of the ARGOS processing chain.

VI. ACKNOWLEDGMENTS

CLS extends a great deal of thanks to the users who kindly authorized access to their data so the new method could be qualified. Their generosity has made it possible for the entire community of ARGOS users to benefit from these positioning improvements. We also welcome any comments from ARGOS users to enhance the capability of the new location algorithm.

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TABLE VI

MEAN ERRORS AND STANDARD DEVIATIONS FOR 1-MESSAGE LOCATIONS. THESE VALUES ARE COMPARED TO THE ERRORS PRODUCED BY THE LEAST SQUARES METHOD WITH 2 OR 3 MESSAGES TO COMPUTE THE VARIATIONS

	Mean errors		Standard deviations	
	Kalman (m)	**Var. (%)**	Kalman (m)	**Var. (%)**
Marabout	17385	**117**	33086	**415**
Geese	16013	**-6**	21672	**-6**
Gnu	2591	**-42**	3838	**-39**
Bighorn	4146	**57**	4844	**38**
Flatback Turtle	3028	**-46**	2254	**-65**
Green Turtle	3176	**-5**	2760	**-33**
Galapagos Sea Lion	5523	**-4**	6585	**12**
Elephant Seal	10868	**-43**	9868	**-35**
Ship	3622	**-6**	4189	**-2**
Drifter	1307	**9**	992	**-13**

TABLE VII

MEDIAN OF THE TANGENTIAL ERROR IN METER WITH A SINGLE MODEL OF RANDOM WALK

	4 messages or more		2 and 3 messages		1 message
	LS (m)	KF (m)	LS (m)	KF (m)	KF (m)
Marabout	21	-28	-370	-18	-125
Geese	-25	-23	-67	-148	42
Gnu	2	2	-72	-112	-304
Bighorn	-9	-6	79	-59	-99
Flatback Turtle	-32	-113	-68	-340	-855
Green Turtle	63	11	88	-493	-1592
Galapagos Sea Lion	-20	-39	122	-7	-186
Elephant Seal	112	-139	-127	-4144	-15028
Ship	0	-19	15	-339	-1618
Drifter	0	-60	19	-363	-1116

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TABLE VIII

MEDIAN OF THE TANGENTIAL ERRORS WITH A BANK OF MODELS INCLUDING BOTH A RANDOM WALK AND A BIASED RANDOM WALK

	4 messages or more		2 and 3 messages		1 message
	LS (m)	KF (m)	LS (m)	KF (m)	KF (m)
Marabout	21	-7	-370	109	13
Geese	-25	-21	-67	-145	-66
Gnu	2	-2	-72	-91	-608
Bighorn	-9	-7	79	-31	21
Flatback Turtle	-32	-70	-68	-147	-418
Green Turtle	63	42	88	-80	-312
Galapagos Sea Lion	-20	-29	122	-10	-38
Elephant Seal	112	12	-127	-217	-1020
Ship	0	5	15	-22	-68
Drifter	0	-30	19	-169	-521